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Holography and Non-Locality in a Closed Vacuum-Dominated Universe

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A closed vacuum-dominated Friedmann universe is asymptotic to a de Sitter space with a cosmological event horizon for any observer. The holographic principle says the area of the horizon in Planck units determines the maximum number of bits of information about the universe that will ever be available to any observer. The wavefunction describing the probability distribution of mass quanta associated with bits of information on the horizon is the boundary condition for the wavefunction specifying the probability distribution of mass quanta throughout the universe. Local interactions between mass quanta in the universe cause quantum transitions in the wavefunction specifying the distribution of mass throughout the universe, with instantaneous non-local effects throughout the universe.

KEY WORDS: holography; non-locality; quantum cosmology; quantized mass.

It now seems inescapable that quantum mechanics is fundamentally non-local. However, a mechanism for non-locality (Einstein's "spooky action at a distance") has been lacking. This paper suggests that the holographic principle indicates a possible mechanism for non-locality in any closed vacuum-dominated Friedmann universe. Specifically, a holographic non-local quantum-mechanical description can be developed for the finite amount of information in a closed vacuum-dominated universe. It is assumed the universe began by a quantum fluctuation from nothing, underwent inflation and became so large that it is locally almost flat. It is also assumed that, after inflation, the vacuum energy density of the universe is constant in space and time (i.e., there is a cosmological constant). One way such a universe can arise is outlined in the quantum cosmology presented in Mongan (2001). When it began, the closed universe contained all the information it will ever contain. There is nothing outside a closed universe, so no information can come into the universe from elsewhere.

At late times in a vacuum-dominated universe, the Friedmann equation becomes $(\frac{\dot{R}}{R})^2 = \frac{8\pi G\epsilon_v}{3c^2} = \frac{\Lambda c^2}{3}$, where the cosmological constant Λ is related to the

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vacuum energy density by $\Lambda = \frac{8\pi G\epsilon_v}{c^4}$, and the universe is asymptotic to a flat de Sitter space. There is a cosmological event horizon in de Sitter space, at a radial distance $R_F = c/H = \sqrt{3/\Lambda}$ from any observer (see, e.g., Carniero (2003); Padmanabhan (2002); Frolov and Kofman (2003)). Taking the Hubble constant as $H_0 = 65\,\text{km}\,\text{sec}^{-1}\,\text{Mpc}^{-1}$, the critical density $\rho_c = \frac{3H_0^2}{8\pi G} = 7.9 \times 10^{-30}\,\text{g}\,\text{cm}^{-3}$, the vacuum energy density $\epsilon_v = 0.7\rho_c c^2 = 5.0 \times 10^{-9}$ g cm^2 sec^{-2} cm^{-3}, and $\Lambda = 1.0 \times 10^{-56}$ cm^{-2}. Then the de Sitter horizon radius is $R_F = 1.7 \times 10^{28}$ cm and the area of any observer's de Sitter horizon is $A = 4\pi R_F^2 = 12\pi/\Lambda$.

Black hole thermodynamics led to the holographic principle (Bousso, 2002), indicating that the number of bits of information inside a horizon with surface area A is $N = A/(4 \ln 2)$, where A is measured in Planck units. Applied to the de Sitter horizon, the holographic principle indicates the total number of bits of information that will ever be available to any observer in the universe is $N_F = \frac{A}{4\ln 2} = \frac{\pi R_F^2}{\ln 2} = \frac{3\pi}{\Lambda \ln 2}$ in Planck units. So, $N_F = \frac{\pi R_F^2}{\delta^2 \ln 2} = \frac{3\pi}{\Lambda\delta^2 \ln 2} = 5.2 \times 10^{122}$, where the Planck length $\delta = \sqrt{\frac{\hbar G}{c^3}} = 1.62 \times 10^{-33}$ cm, and N_F is also the number of pixels of area $4\delta^2 \ln 2$ on any observer's de Sitter horizon. Reasonable physical theories must be based only on information available to an observer, and the de Sitter horizon specifies the total amount of information that will ever be available to any observer. Each observer has a different de Sitter horizon, but (consistent with relativity) no observer has a privileged position. When we are eventually able (in principle) to observe the de Sitter horizon, the holographic information on the horizon will characterize the state of the universe at the instant, billions of years previously, when the photons carrying the information left the horizon.

After the first few seconds of the life of the universe, the energy exchanged between matter and radiation was negligible compared to the total energy of matter and radiation separately (Misner *et al.*, 1973, p. 726). In a closed universe, when the exchange of energy between matter and radiation is negligible, the total mass of the universe and the total number of massless quanta in the universe are conserved (Misner *et al.*, 1973). There are about 10^{90} massless quanta in a closed universe with today's radiation energy density, a radius of about 10^{28} cm and a cosmic microwave background temperature of 2.73 °K. This number is negligible compared to the total number of degrees of freedom in the universe, so the following analysis focuses on a quantum mechanical description of the distribution of the mass within the universe and neglects massless degrees of freedom.

Describing a universe with only a finite number of degrees of freedom is an extremely difficult problem for quantum field theory, so this analysis is not based on quantum field theory. A quantum mechanical description of the finite number of bits of information on a horizon requires wavefunctions specifying the probability distribution of those bits of information on the horizon. This analysis considers a description of the universe that can in principle be developed by observers using the finite amount of information available on their de Sitter horizon.

Wesson (2004) notes that two mass scales, a quantum mass scale and a gravitational mass scale can be formed from the constants G,$\hbar$, Λ and c. The quantum mass scale $\frac{\hbar}{c}\sqrt{\frac{\Lambda}{3}} = 2.0 \times 10^{-66}$ g is the scale for the minimum quantum of mass in the universe. The gravitational mass scale $\frac{c^2}{G}\sqrt{\frac{3}{\Lambda}} = 2.3 \times 10^{56}$ g is the scale for the total conserved mass of a closed universe. The holographic principle says there are $\frac{3\pi}{\Lambda\delta^2 \ln 2}$ observable bits of information on the de Sitter horizon at radius $\sqrt{3/\Lambda}$ in a universe dominated by a cosmological constant Λ. If the total conserved mass of the closed universe is $\frac{bc^2}{G}\sqrt{\frac{3}{\Lambda}}$, and there are $\frac{3\pi}{\Lambda\delta^2 \ln 2}$ bits of information on the de Sitter horizon, the mass associated with each bit is $(\frac{bc^2}{G}\sqrt{\frac{3}{\Lambda}})/(\frac{3\pi}{\Lambda\delta^2 \ln 2}) = \frac{b \ln 2}{\pi}\frac{\hbar}{c}\sqrt{\frac{\Lambda}{3}}$. So, the holographic principle indicates that the mass scale for a bit of information is the quantum mass scale $\frac{\hbar}{c}\sqrt{\frac{\Lambda}{3}} = 2.0 \times 10^{-66}$ g for the minimum quantum of mass in the universe. The quantum states with the longest wavelength $2\pi R_F = 2\pi\sqrt{3/\Lambda}$ correspond to states with the lowest energy on the horizon and should be associated with the quantum of mass. If the quantum of mass is $\frac{\hbar}{c}\sqrt{\frac{\Lambda}{3}}$, that mass quantum has Compton wavelength $2\pi R_F$. This suggests that $\frac{b \ln 2}{\pi} = 1$ and each bit of information is associated with a quantum of mass $\frac{\hbar}{c}\sqrt{\frac{\Lambda}{3}}$ with Compton wavelength $2\pi R_F$.

As noted previously, the number of bits of information in the universe $N_F = \frac{4\pi R_F^2}{4\delta^2 \ln 2} = 5.2 \times 10^{122}$ is also the number of pixels of area $4\delta^2 \ln 2$ on any observer's de Sitter horizon. So, a quantum description of the total amount of information available about the universe can be obtained by identifying each area (pixel) of size $4\delta^2 \ln 2$ on the de Sitter horizon with one bit of information, associated with the wavefunction for a quantum of mass $\frac{\hbar}{c}\sqrt{\frac{\Lambda}{3}}$ with Compton wavelength $2\pi R_F$. If the z axis (in spherical coordinates centered on the observer's position) pierces the center of the ith pixel of area $4\delta^2 \ln 2$ on the horizon with radius R_F, θ_i is the polar angle measuring the angular distance from the z axis through the ith pixel to the point on the horizon where the wavefunction is evaluated and C_2 is a normalization constant, wavefunctions of the form $C_2 \cos\theta_i$ have the necessary wavelength $2\pi R_F$ and can define the probability for finding the bit of information associated with the ith pixel at any location on the horizon. The chosen form of the wavefunction $C_2 \cos\theta_i$ insures that the maximum probability of finding the ith bit of information on the de Sitter horizon is in the two pixels on opposite hemispheres of the horizon where the z axis of that wavefunction pierces the horizon. These wavefunctions have anti-nodes where the z axis intercepts the horizon and a nodal line around the equator (polar angle = $\pi/2$).

Regarding the wavefunctions associated with two diametrically opposite pixels, if $+1$ is assigned to pixels containing a "peak" anti-node and -1 is assigned to pixels containing a "valley" anti-node, two situations are possible. If two pixels diametrically opposite each other on different hemispheres of the horizon have the same sign, the probability waves destructively interfere all over the surface, resulting in zero probability of finding mass quanta associated with a bit of information at any location on the horizon. If two diametrically-opposed pixels have opposite signs, the wavefunctions constructively interfere, resulting in a wavefunction $\psi_i = \sqrt{\frac{3}{2\pi R_F^2}} \cos\theta_i$ normalized to two mass quanta on the horizon. In this way, the constructive and destructive interference of the $\cos\theta_i$ wave functions on the horizon requires that mass quanta occur in pairs. So no additional bits of information are required to encode the information in the universe if each member of a pair of mass quanta has the opposite value of a single (conserved) quantum number.

The quantum of mass $\frac{\hbar}{c}\sqrt{\frac{\Lambda}{3}}$ is far below the mass scales familiar from experimental particle physics. The proton mass, $m_p = 1.67 \times 10^{-24}\,\text{g} = 938\,\text{MeV}$ is about 10^{42} times the quantum of mass and the electron mass 0.511 Mev is about 5×10^{38} times the quantum of mass. A mass as small as 1 eV is about 10^{33} times the quantum of mass, and the current upper limit on the mass of the photon is about fifteen orders of magnitude larger than the mass quantum. So, there are at present no theories relating the quantum of mass to the familiar masses observed in particle physics experiments. However, if each member of a pair of mass quanta has opposite values of a single as yet unknown quantum number, composite massive systems described by a superposition of wavefunctions for pairs of mass quanta need not be identical except for their mass. A theory involving mass quantum pairs with opposite values of a single quantum number may allow properties like spin and charge to emerge as quantum numbers characterizing composite quantum mechanical systems involving the large numbers of mass quanta necessary to make the "elementary" particles we have observed to date.

The distribution of mass quanta associated with bits of information on the horizon at any instant of cosmic time t can be represented by a two-dimensional lattice gas involving $\frac{N_F}{2}$ mass quanta with effective mass $\frac{2\hbar}{c}\sqrt{\frac{\Lambda}{3}}$ where each lattice gas site has area $4\delta^2 \ln 2$. The lattice gas has zeros in half of the pixels on the horizon and ones in the remaining pixels. Pixels diametrically opposite to each other on the horizon are constrained to have the same value. Zeros correspond to situations where wavefunctions from diametrically-opposed pixels on the entire horizon destructively interfere to produce zero probability of finding mass quanta associated with bits of information related to those pixels at any location on the horizon. The remaining pixels, with a value 1, contain an anti-node of the wavefunction $\psi_i = \sqrt{\frac{3}{2\pi R_F^2}} \cos\theta_i$, normalized to two mass quanta on the horizon of radius R_F.

The probable number n of the mass quanta associated with one bit of information in an area A on the horizon is $n(A) = N_F \iint_A |\prod_{i=1}^{N_F} a_i \psi_i|^2 \; dA$, based on the lattice gas representation of the information on the horizon. This is also the probability of finding $n(A)$ of the mass quanta associated with a bit of information in the solid angle within the universe that subtends the area A on the horizon and has its apex at the observer's position. The N_F numbers a_i associated with each pixel on the horizon are either zero or one, the numbers corresponding to pixels diametrically opposite to each other on the horizon are constrained to be equal, and together these numbers encode all the information that will ever be available about the universe.

The wavefunctions on the horizon are the boundary condition on the form of the wavefunctions specifying the probability distributions of the finite number of mass quanta distributed throughout the featureless background space of a closed universe with a constant vacuum energy density (cosmological constant). The wavefunction for the probability of finding a mass quantum anywhere in the universe is a solution to the Helmholtz wave equation in the closed universe. A closed universe with radius of curvature R can be defined by the three coordinates χ, θ and ϕ, where the volume of the three-sphere (S^3) is $R^3 \int_0^{\pi} \sin^2\chi \, d\chi \int_0^{\pi} \sin\theta \, d\theta \int_0^{2\pi} d\phi$ (Islam). The wavefunction for a bit of information on the horizon is the projection of the wavefunction for that bit of information in the volume of the universe on the horizon at $R_F = R\sin\chi$. One solution to the Helmholz equation on the three-sphere (Gerlach and Sengupta, 1978) is the scalar spherical harmonic $Q_{10}^2 = \sqrt{\frac{12}{\pi}} \cos\theta_i \; \sin\chi$. For the ith degree of freedom on the horizon, the $\cos\theta_i$ behavior of the wavefunction on the horizon determines the wavefunction $\Psi_i = C_3 \, Q_{10}^2(i) = C_3\sqrt{\frac{12}{\pi}} \cos\theta_i \sin\chi$ as the solution of the Helmholz wave equation describing the probability distribution within the universe of the mass quanta associated with the ith bit of information. The constant $C_3 = \sqrt{\frac{4}{\pi^2 R^3}}$ is determined by normalizing the wavefunction to two quanta of mass within the universe. The probable number n of the mass quanta associated with one bit of information in any volume V within the universe is $n(V) = N_F \iiint_V |\prod_{i=1}^{N_F} a_i \Psi_i|^2 \, dV$. So, the information specified by the finite quantum lattice gas representation on the horizon determines the probability of finding a certain number of mass quanta in any given volume within the universe.

Quantum state changes caused by local interactions between mass quanta have nonlocal consequences throughout the universe. Any local change in the quantum state of the mass distribution within the universe is instantly reflected in changes in the eigen-solutions of the Helmholtz wave equation within the universe, as well as in the lattice gas representation of the universe on an observer's de Sitter horizon. This instantly changes both the probability distribution of the bits of information on the horizon and the corresponding probability distribution of mass quanta throughout the universe. In this way, the holographic principle indicates a mechanism for non-locality in quantum processes throughout the universe.

If the universe is closed, it is now so large that it is locally flat. At present, we can't see beyond the surface of last scattering that displays information from the era when photons decoupled from matter and began streaming freely through the universe. However, as we look out into the universe, Bousso's light sheet analysis (Bousso, 2002) indicates that data from observations at redshift z, corresponding to a proper distance D from us, can be used to develop a holographic description of the universe at the corresponding lookback time. Because the area of the surface at distance D is smaller than the area of the de Sitter horizon, the description will be more coarse grained than the description based on information from the deSitter horizon. The light sheet analysis indicates that the number of bits of information available to any observer on the spherical surface at distance D from us is $N_E = \frac{\pi D^2}{\delta^2 \ln 2} = \frac{\pi c^3 D^2}{\hbar G \ln 2}$. The mass associated with each bit of information is then $(\frac{\pi c^2}{G \ln 2}\sqrt{\frac{3}{\Lambda}})(\frac{\hbar G \ln 2}{\pi C^3 D^2}) = \frac{\hbar}{c}(\frac{\sqrt{3/\Lambda}}{D^2})$, and this is larger than the quantum of mass $\frac{\hbar}{c}\sqrt{\frac{\Lambda}{3}}$ until $D = R_F = \sqrt{3/\Lambda}$. The holographic principle again indicates the probability distribution of mass within the universe is encoded by a lattice gas representation involving N_E sites on the spherical surface at distance D from the observer. The number of bits of information available for describing the probability distribution of mass within the universe goes up and the average mass associated with those bits of information goes down as the distance D increases, until the de Sitter horizon is reached.

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